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The Propagation of Depletion – The Inclusion of Inertia in the Derivation of the Diffusivity Equation

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Outline

- Intro: Why is this even important?
- Assumptions in the Traditional Solution for the Diffusivity Equation
- Inclusions in the Proposed Solution for the Diffusivity Equation
- Comparison of the Two Equations
- Consequences of the Inclusion of Inertia, Capillary Threshold Pressure and 2nd Law of Thermo Constraints
- Equation for Boundary Dominated Radial Flow
- Case Study/Example
- Conclusions
- Future Work

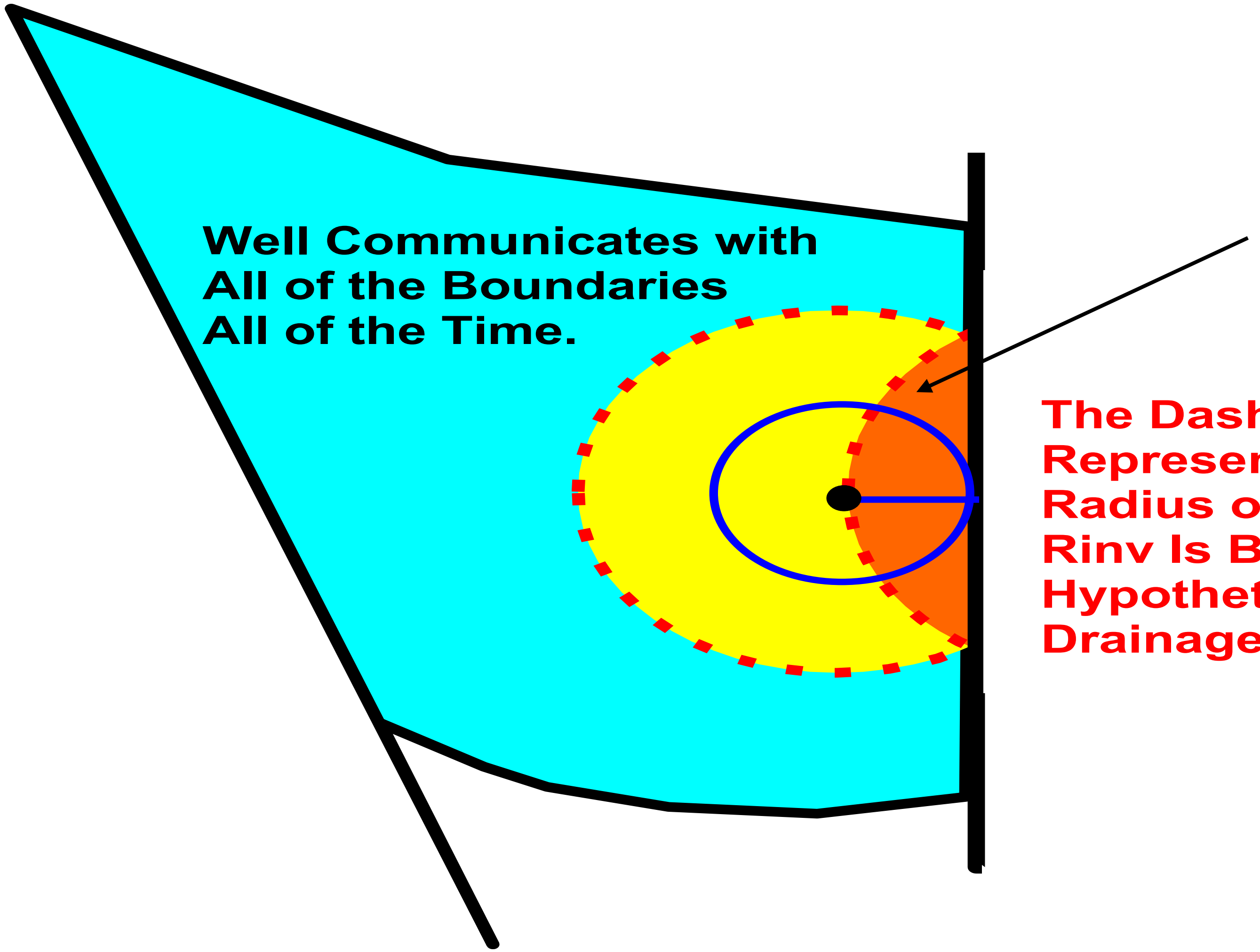
Intro: Oi! What's the Point of This Anyway?

- The Classic Solution to the Diffusivity Equation is Good for IARF at the well, not so good away from the well…not good anywhere between the time of the first boundary contact and the start of some sort of SS flow in the reservoir

- Need an equation to predict the pressure response at any point in the reservoir before SS flow

- Better Well Test Planning
- Better Understanding of Interference/Communication
- Better Visualization of Reality

Traditional/Classic Diffusion Model



Wave “Bounces” Back
Caused by Imaginary Well

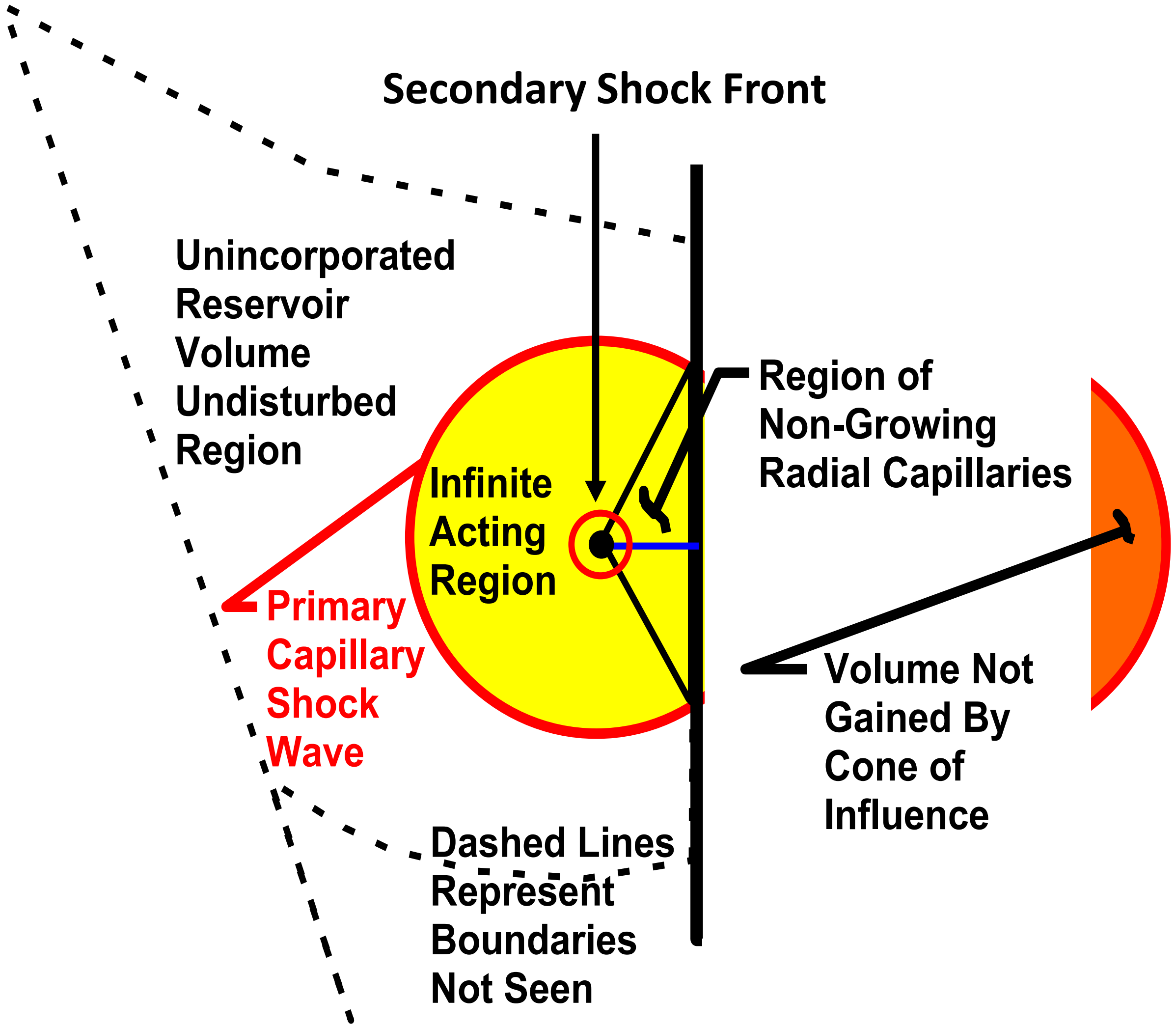
The Dashed Red Line
Represents the
Radius of Investigation.
Rinv Is Based Upon the
Hypothetical Effective
Drainage Volume of the Well..

Traditional Diffusion Model - Assumptions

- Higher order terms are neglected (Inertia, Capillary Pressure & Momentum)
- Zero Potential Flow – An infinitesimal DP Can and Does Cause Flow
- Point to Point Potential Gradient Flow
- Fixed Boundaries as Initial Condition
- The Producing well is in Communication with the Entire Reservoir the Moment it’s Turned on
    - Thermodynamics and The Law of Relativity Apparently Don’t Apply to Petroleum Reservoirs!
- Smooth, Continuous Relaxation Curve in Pressure Response
- Waves Bounce Around Like a Rock in a Swimming Pool

- Conductive Disk Analogy from Heat Transfer Theory (Hurst, 1933)
- …But, This is Mass Transfer in Porous Media, not Heat X-fer in a Solid Chunk of Metal!

Proposed Nested Cone/Regeneration Diffusion Model

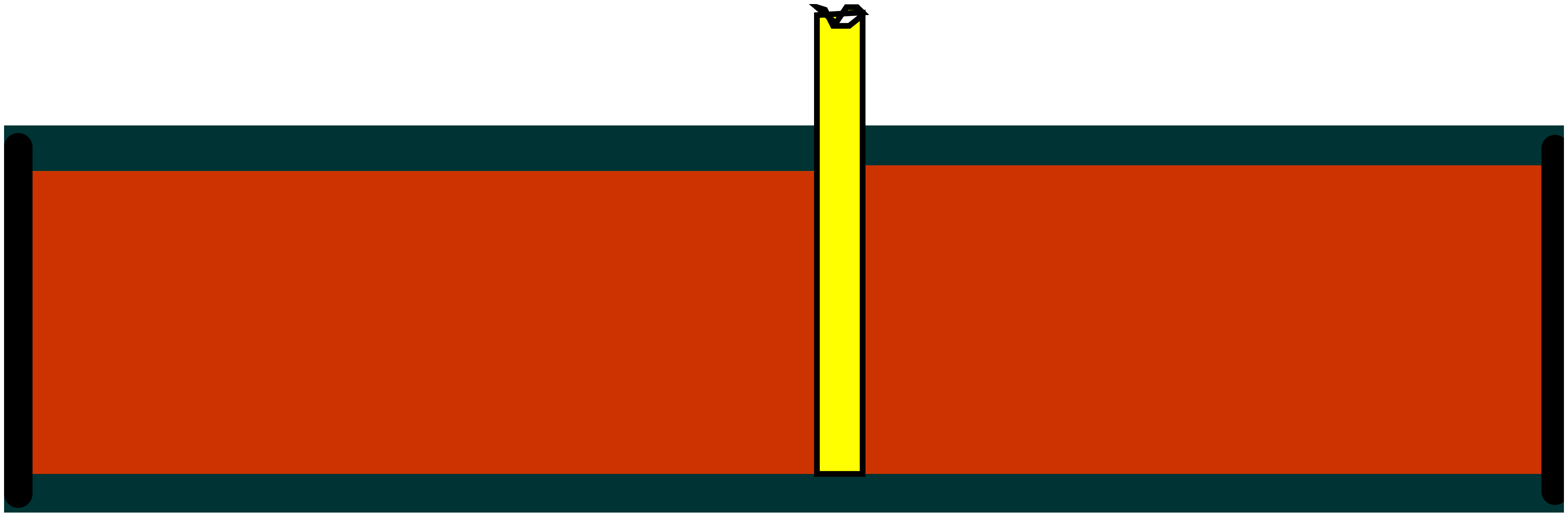


Inclusions in the Proposed Solution for the Diffusivity Equation

- Higher order terms are included (Inertia, Cap Pressure, Momentum, etc.)
- No Zero Potential Flow – An infinitesimal DP Cannot Cause Flow
- Darcy Flow only in Active Capillaries
- Boundaries are not Observed Until the Primary Shock Front Strikes them
- The Producing well is NOT in Communication with the Entire Reservoir the Moment it's Turned on
    - The System Must Obey the Laws of Thermodynamics
- Log-Linear Pressure Response After the Shock Front Arrives
- Shock Front Regeneration, not Reflection

Diffusion Model

All Reservoir Volume Is Active
All Boundaries Influence Model All of the Time



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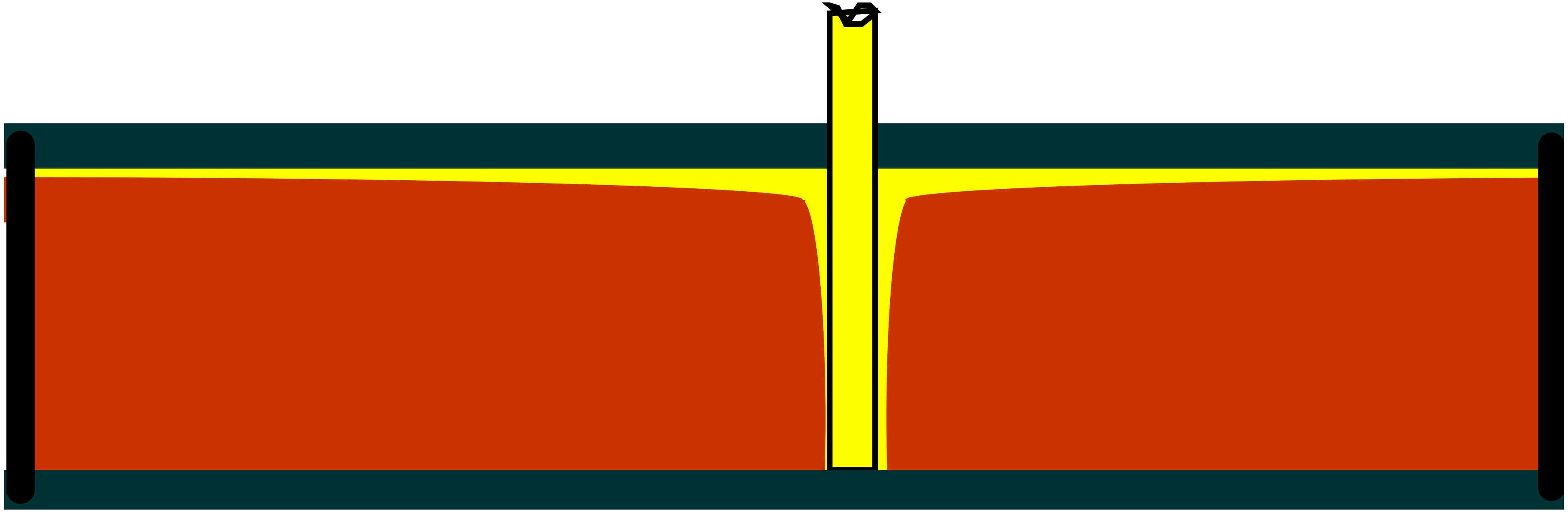

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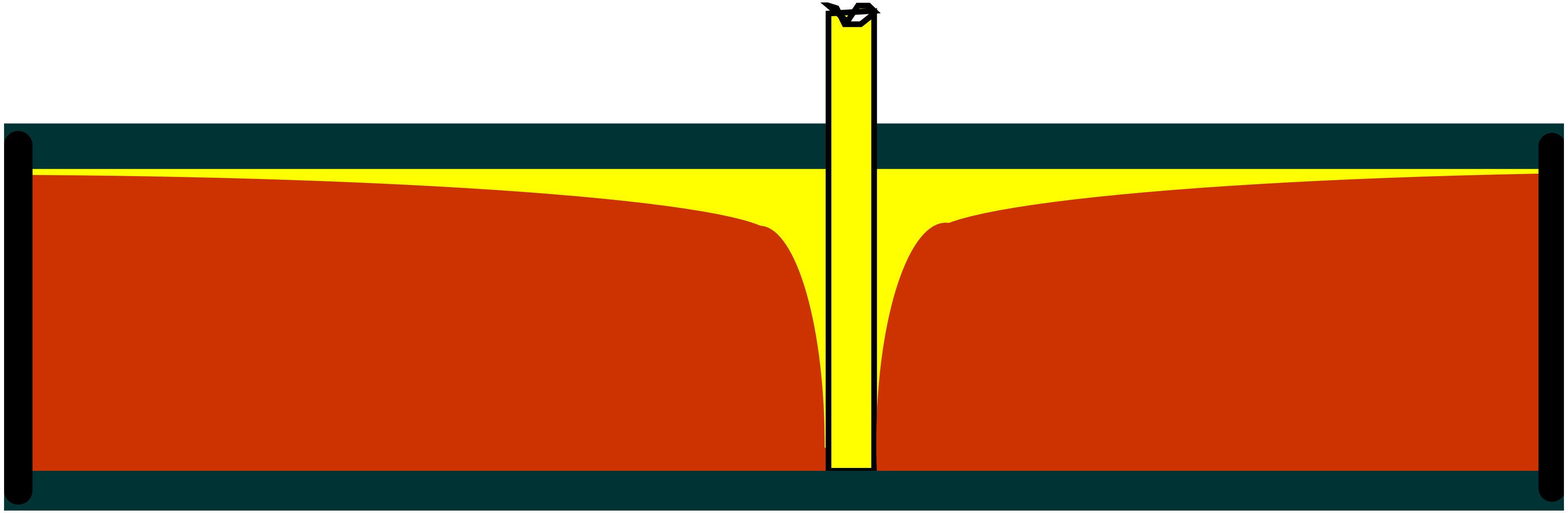

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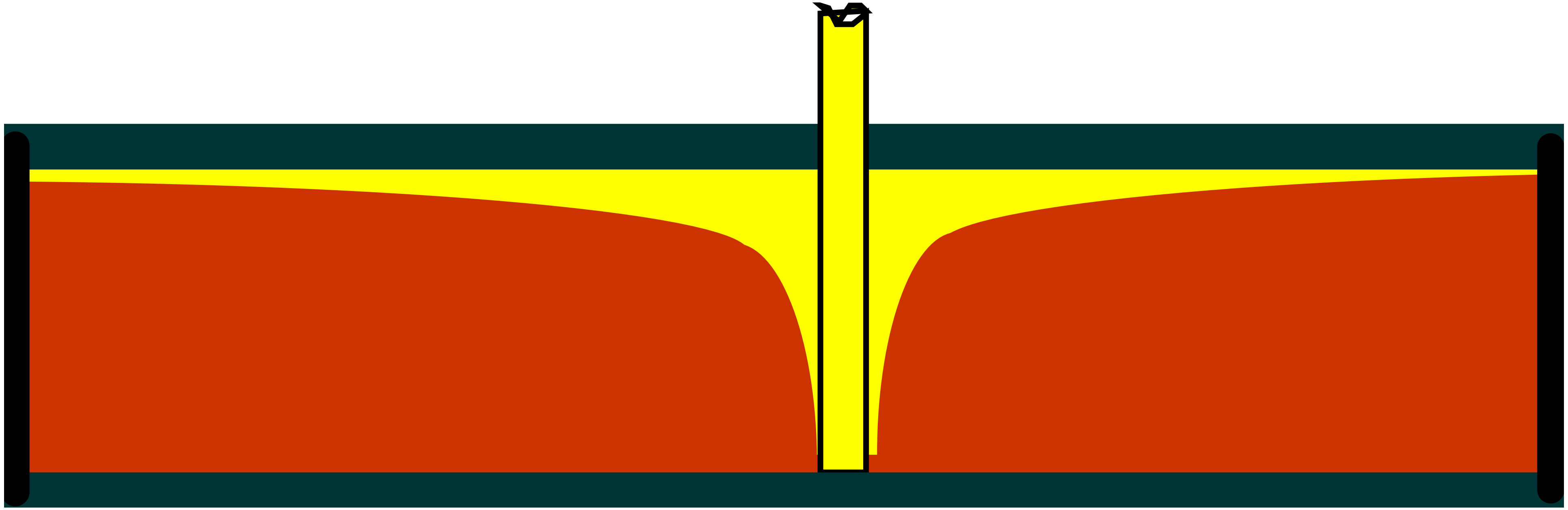

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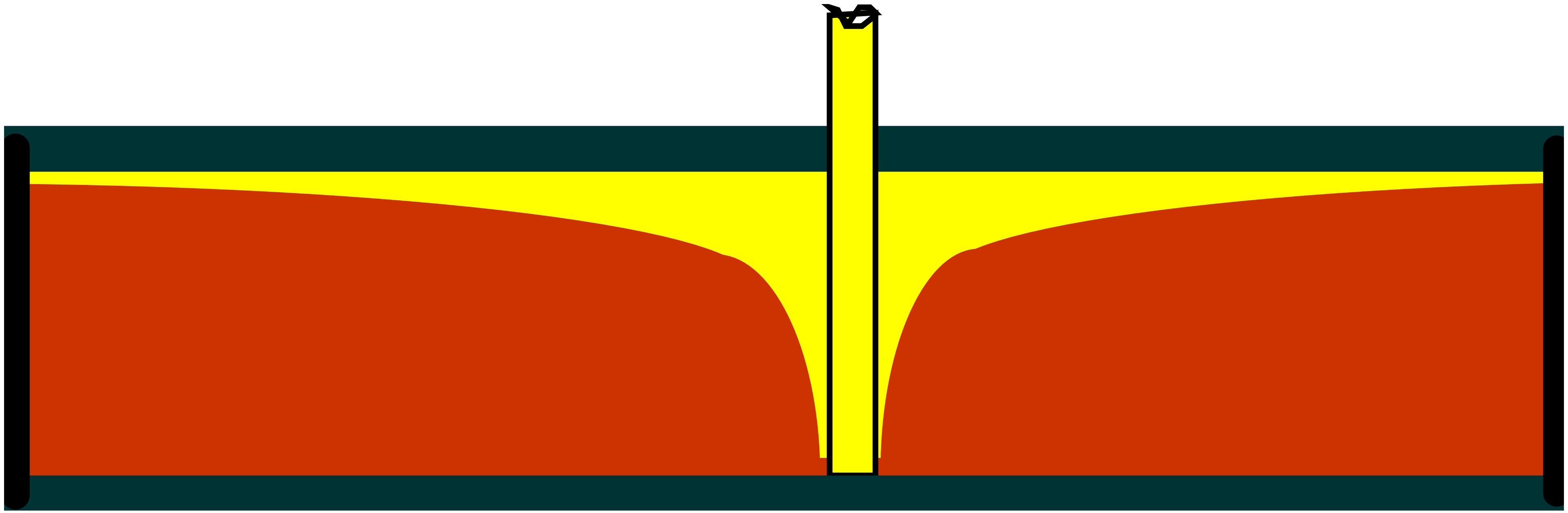

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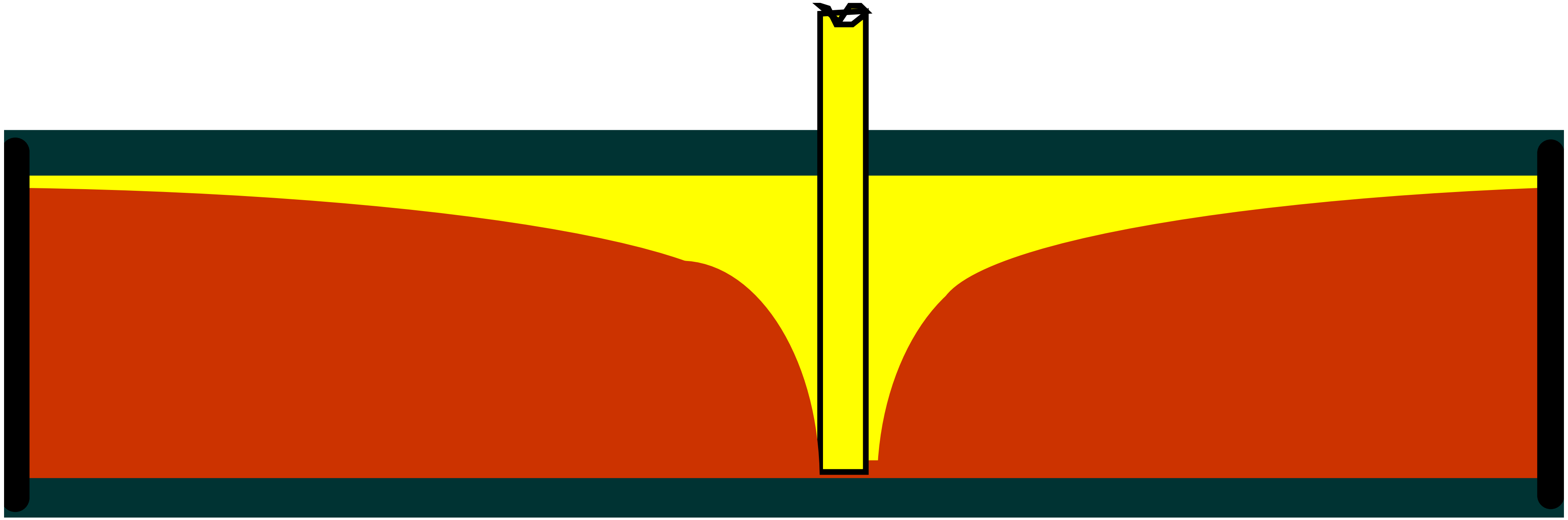

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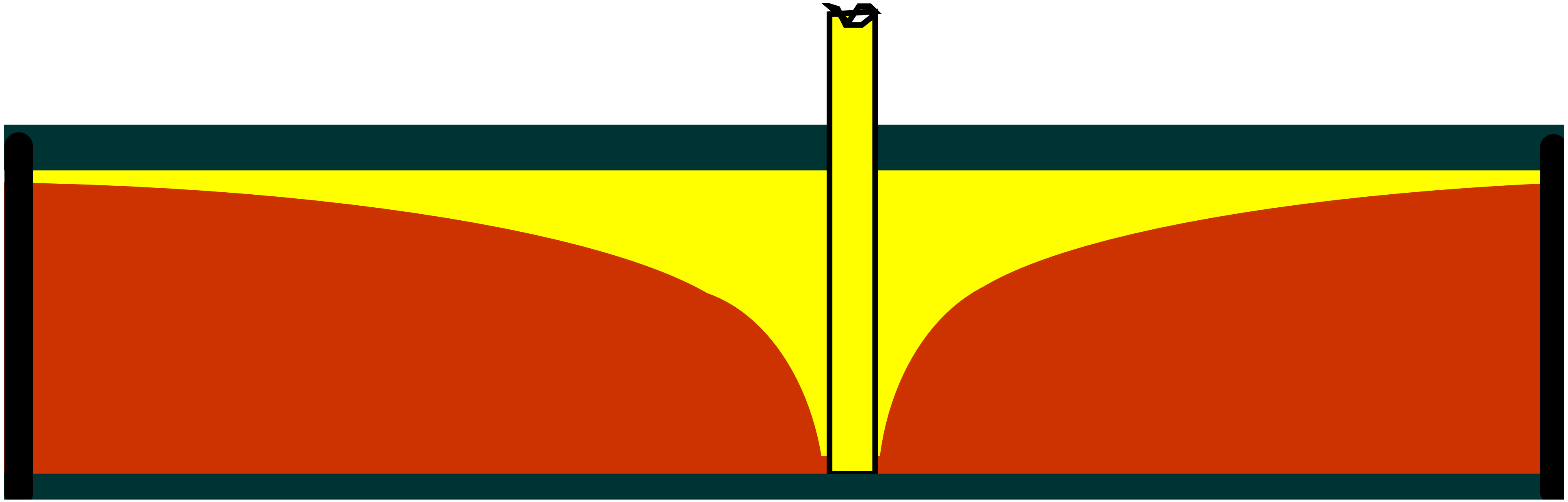

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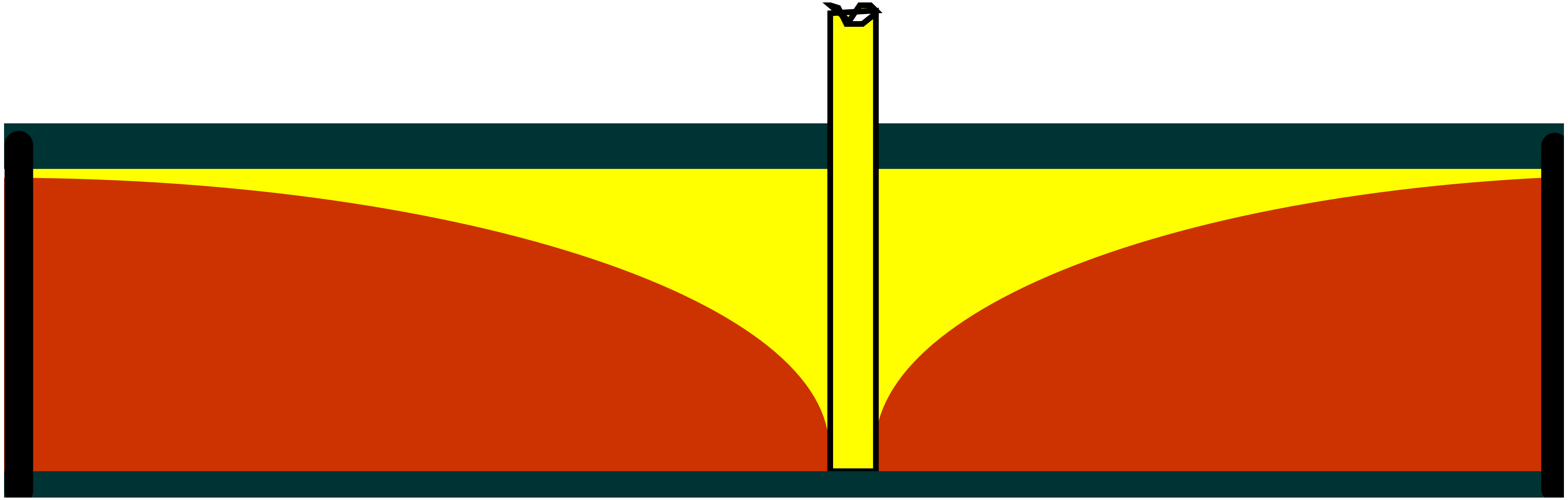

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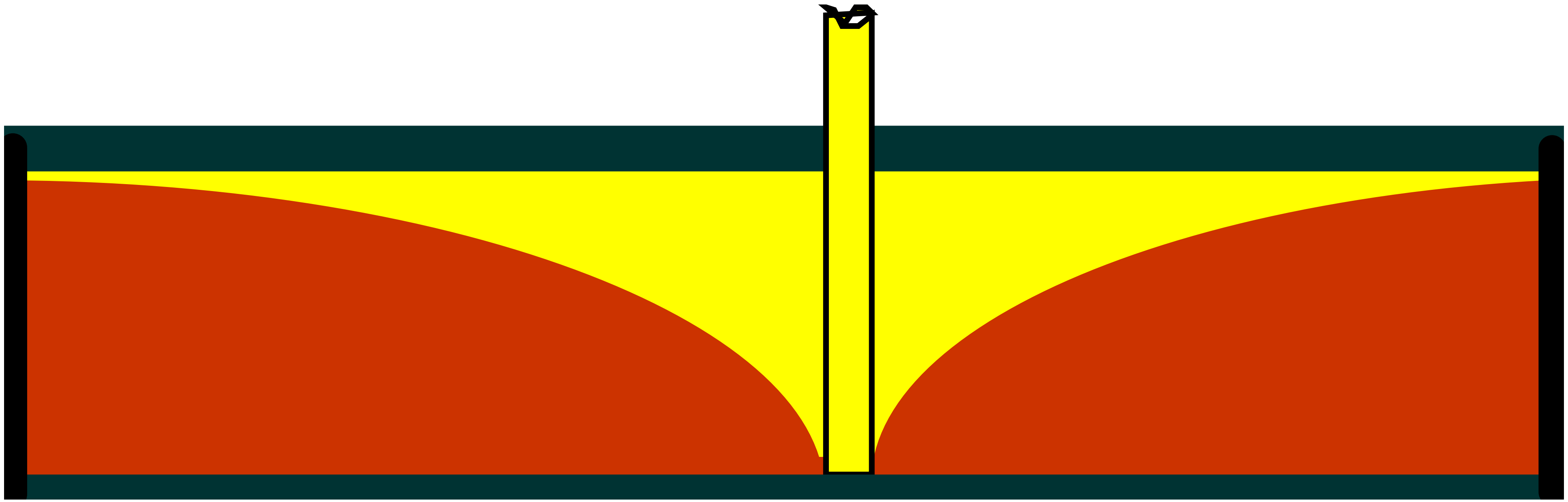
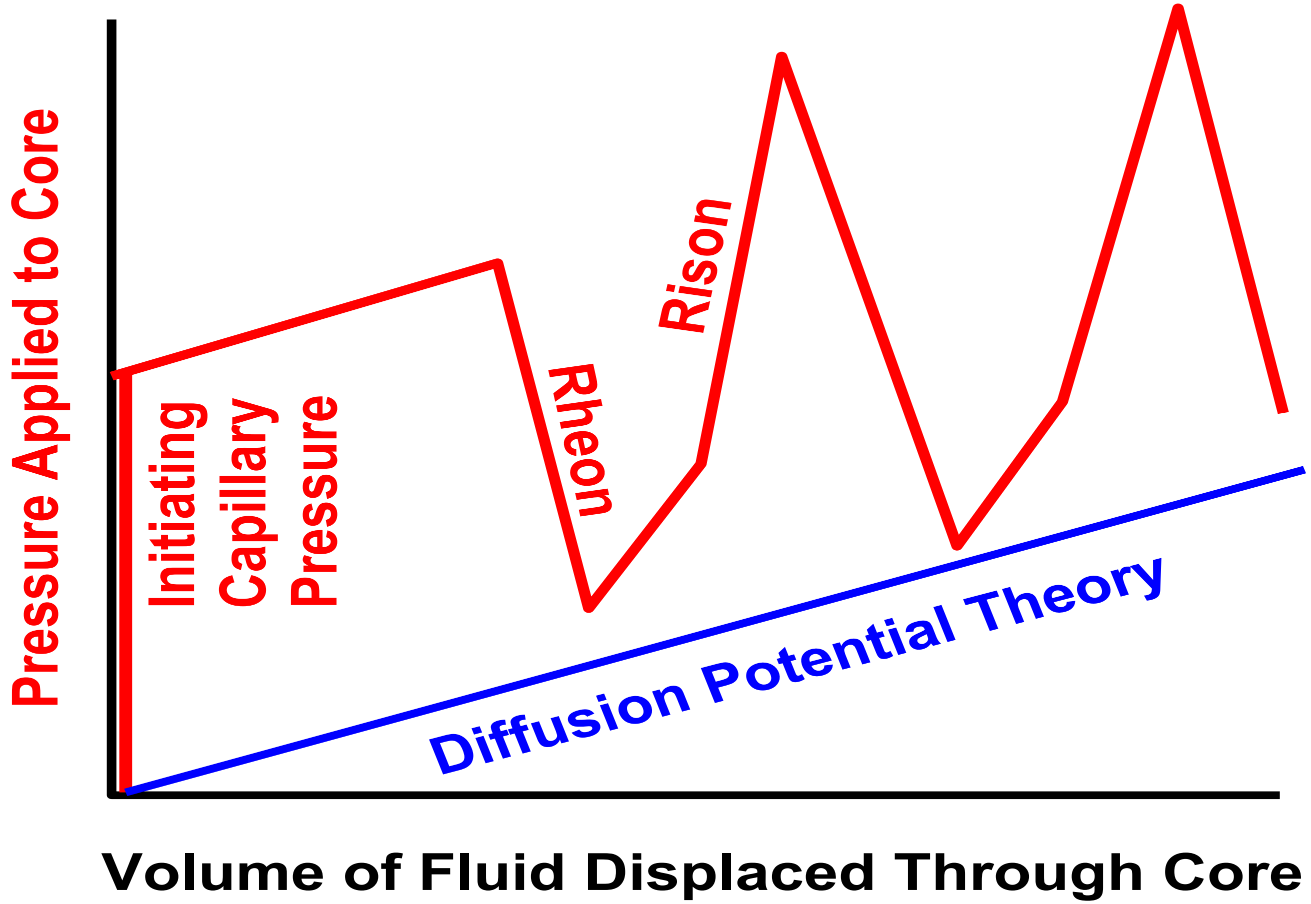


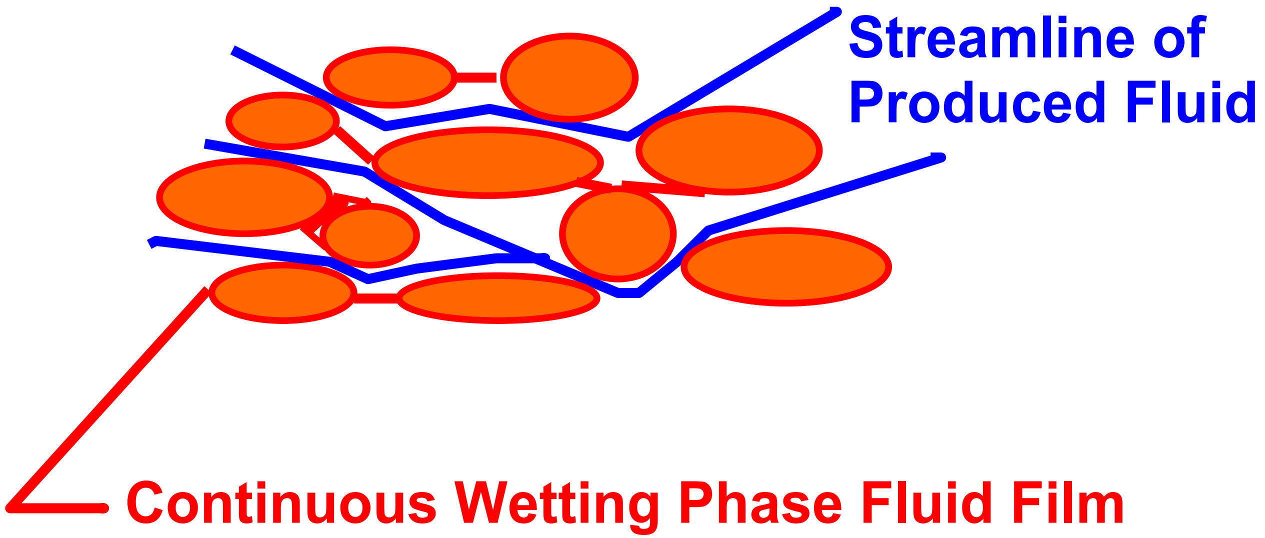
The Proposed Nested Cone/Regeneration Capillary Model

Is Based Upon a Growing Capillary Cluster Radiating From the Producing Well.

Actual Flow Through a Core – Zero Potential Flow Does Not Exist!

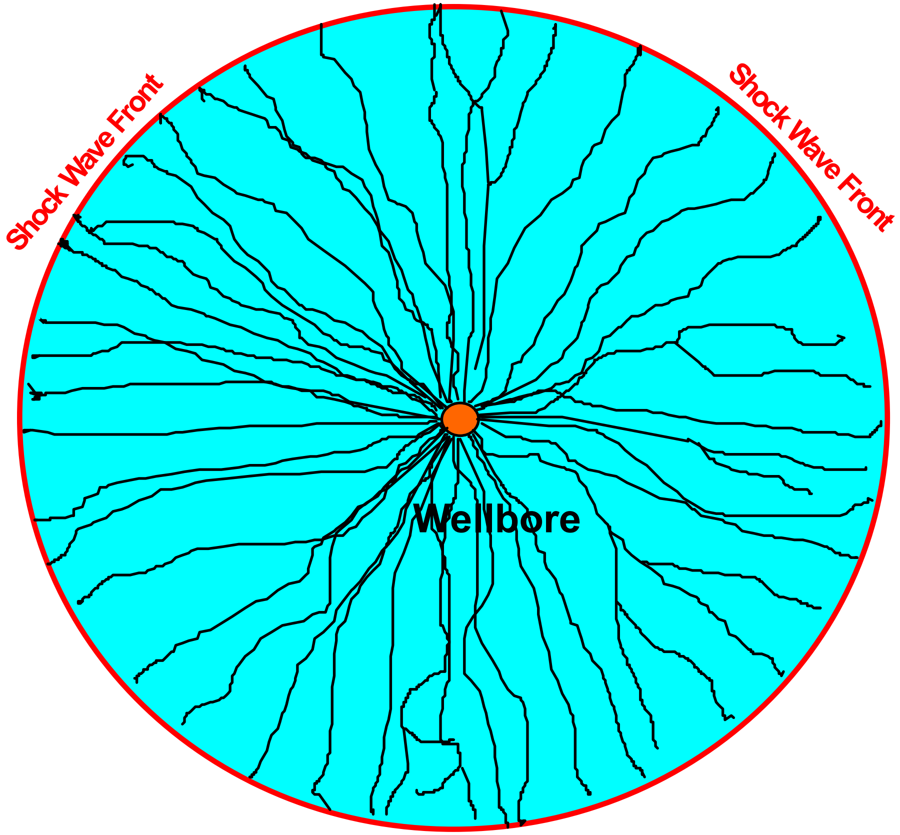


Not All Pore Throats Break…This Creates a Capillary Structure

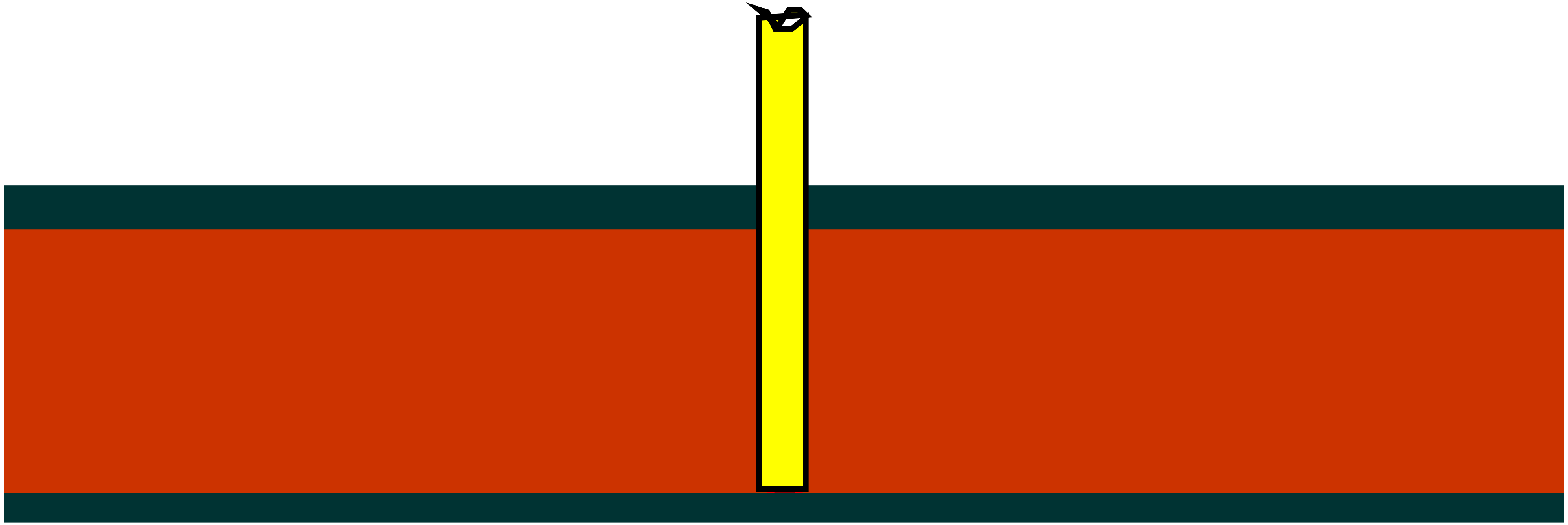


To Break the Fluid Film in Order to Allow a Change in Flow Path, Requires a Finite Initiating Differential Pressure Across any Pore Throat.

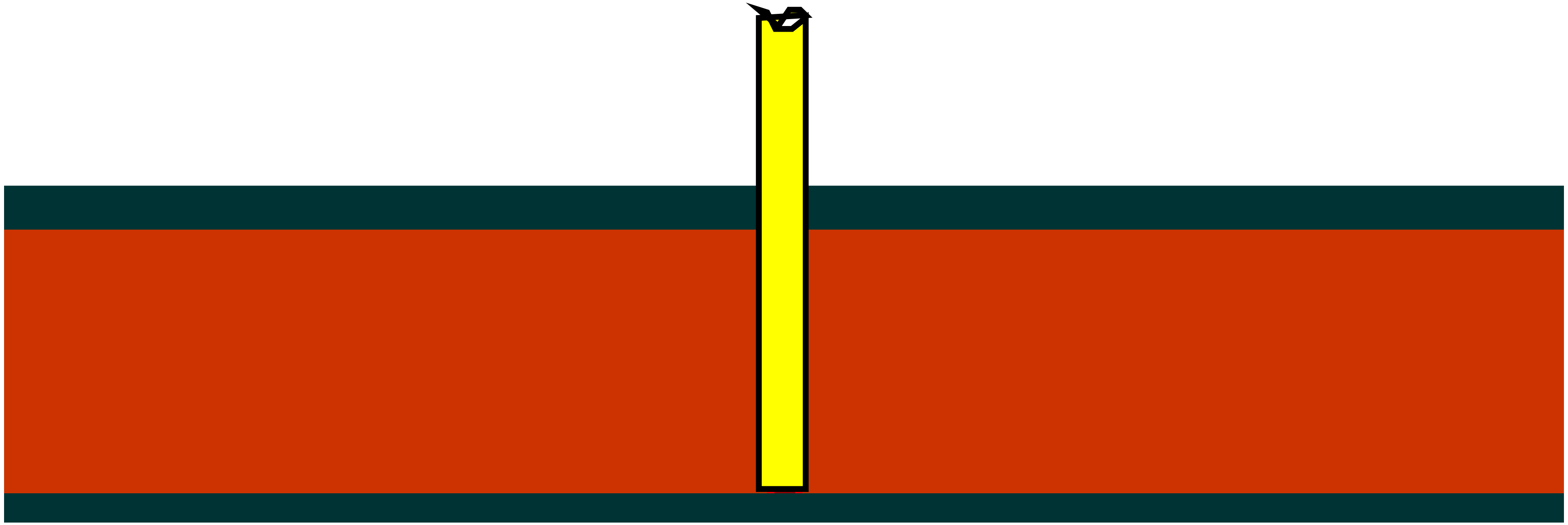
Clusters of Growing Capillaries



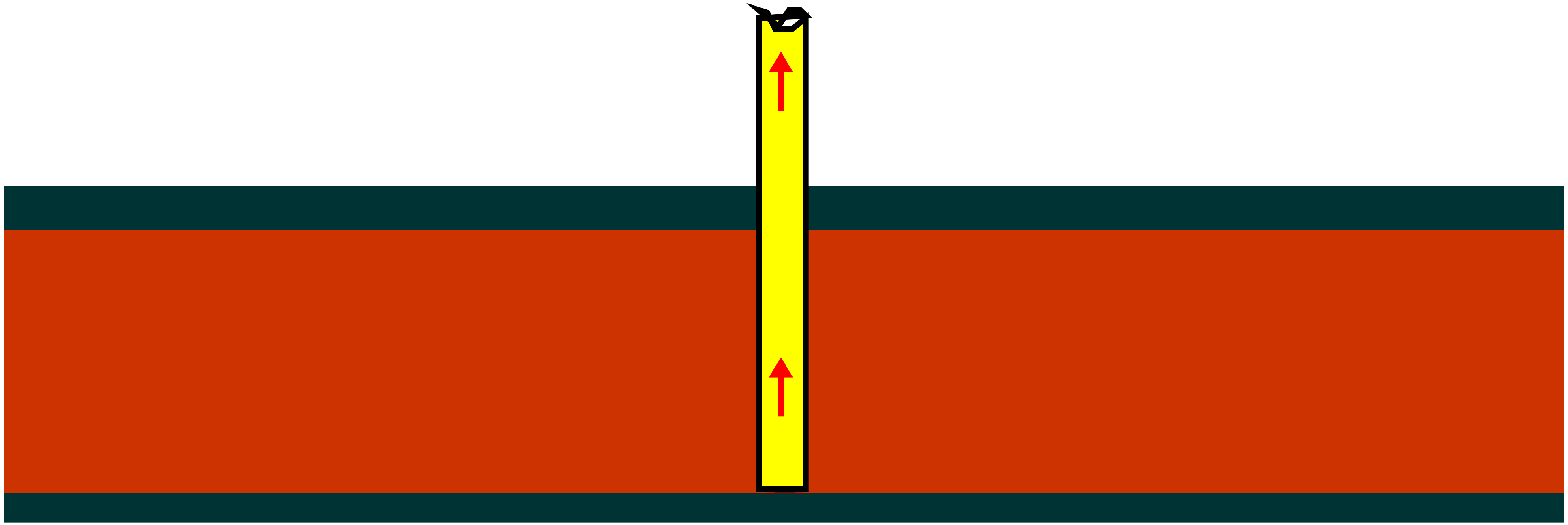
Radial Pressure Influence in Active Reservoir Volume



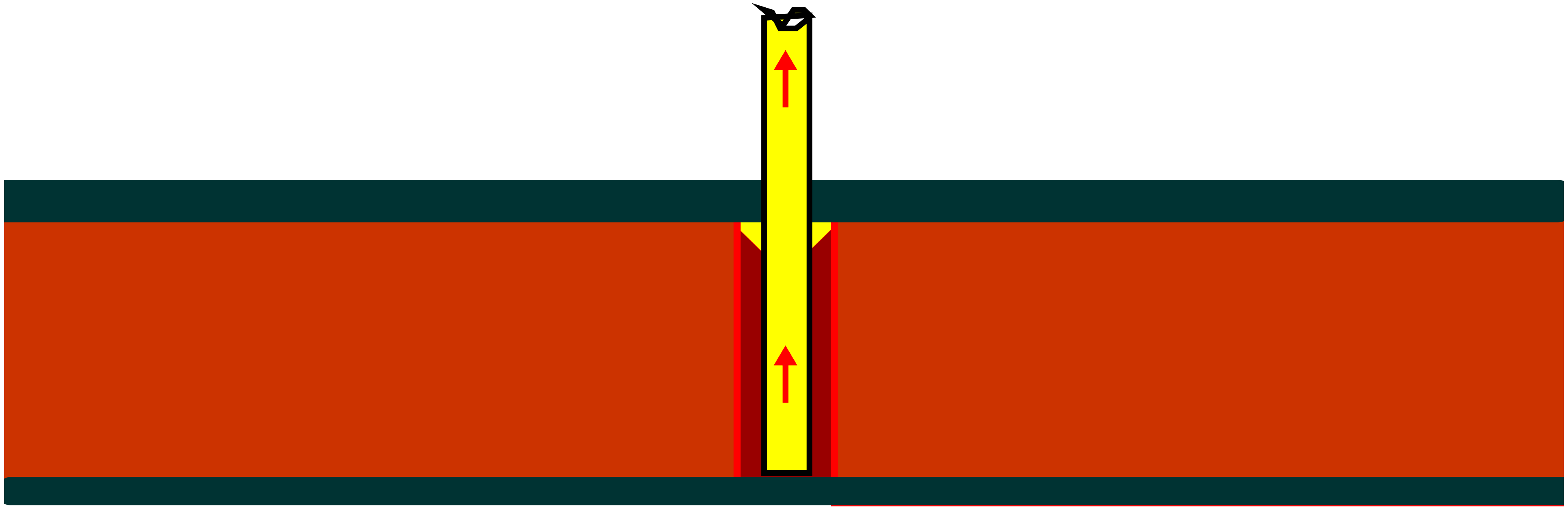
Static Reservoir



Well Turned On



Shock Wave Advances, Creates Active Reservoir Volume; Pressure Depletes Behind Wave



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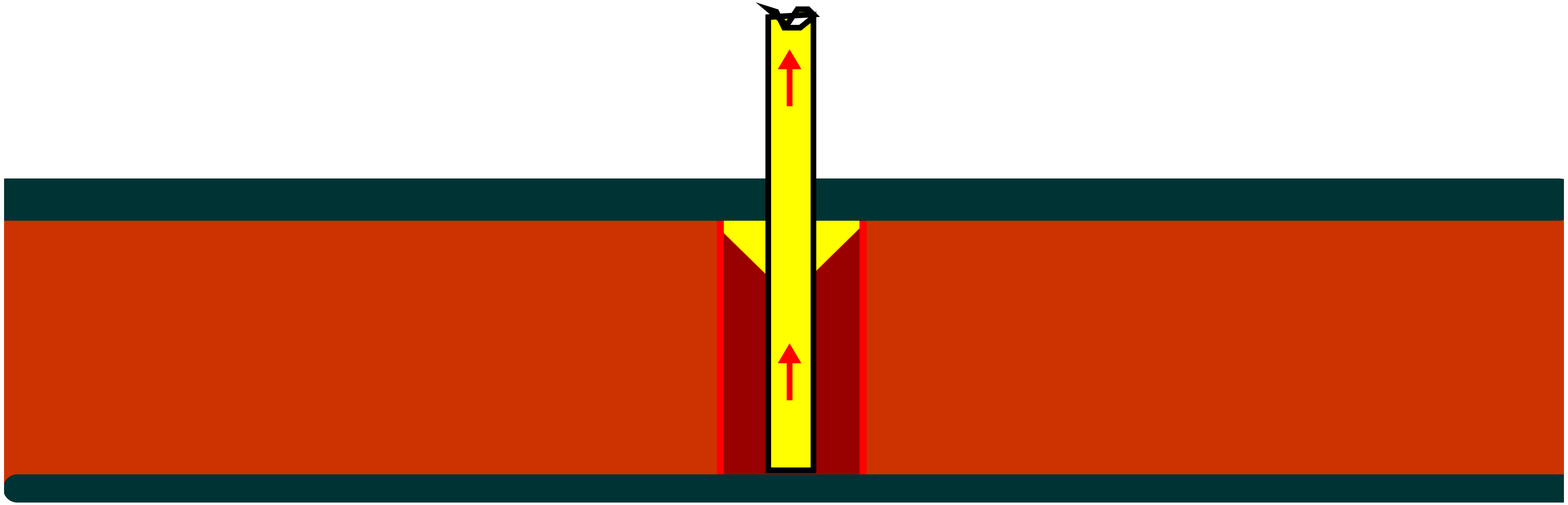

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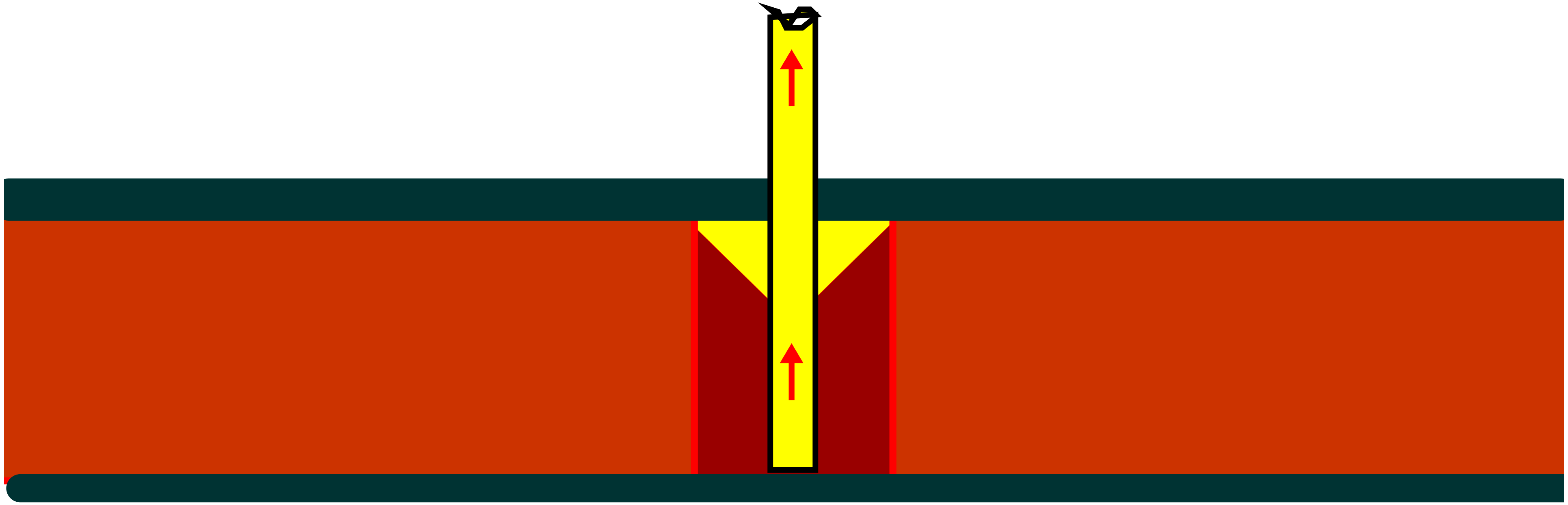

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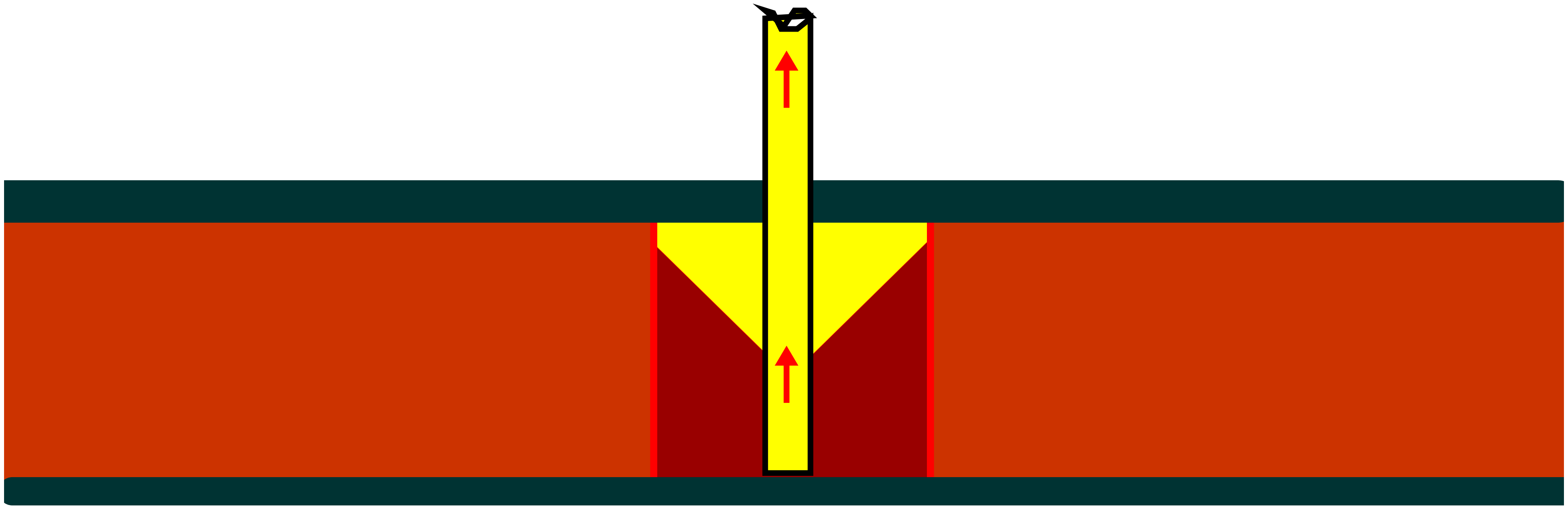

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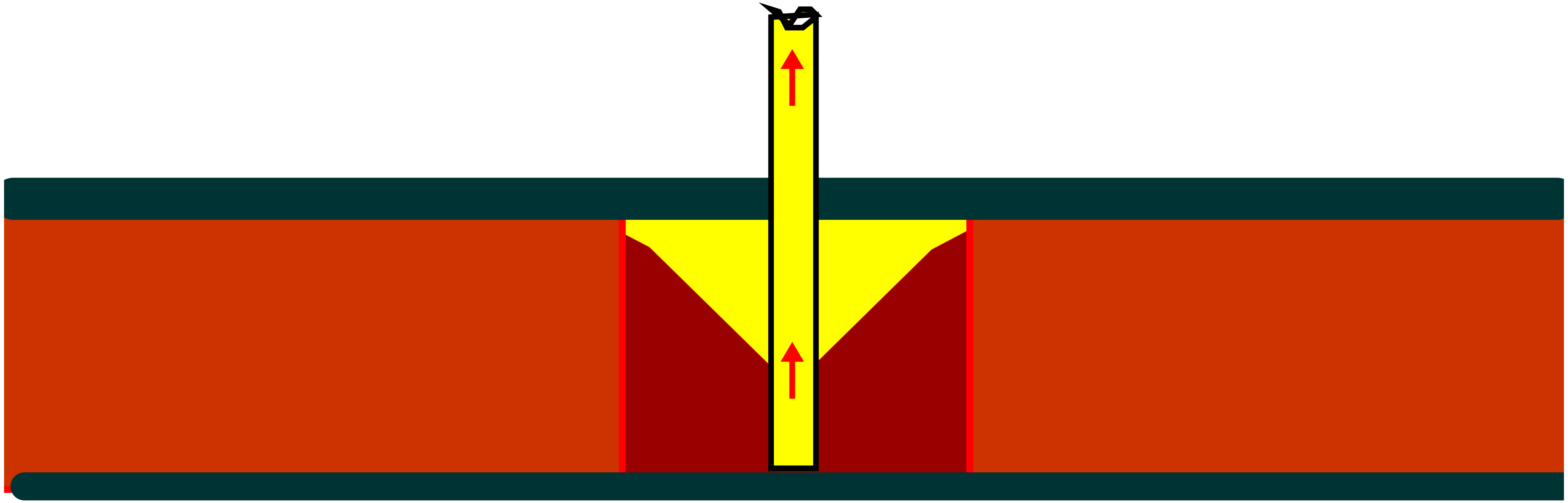

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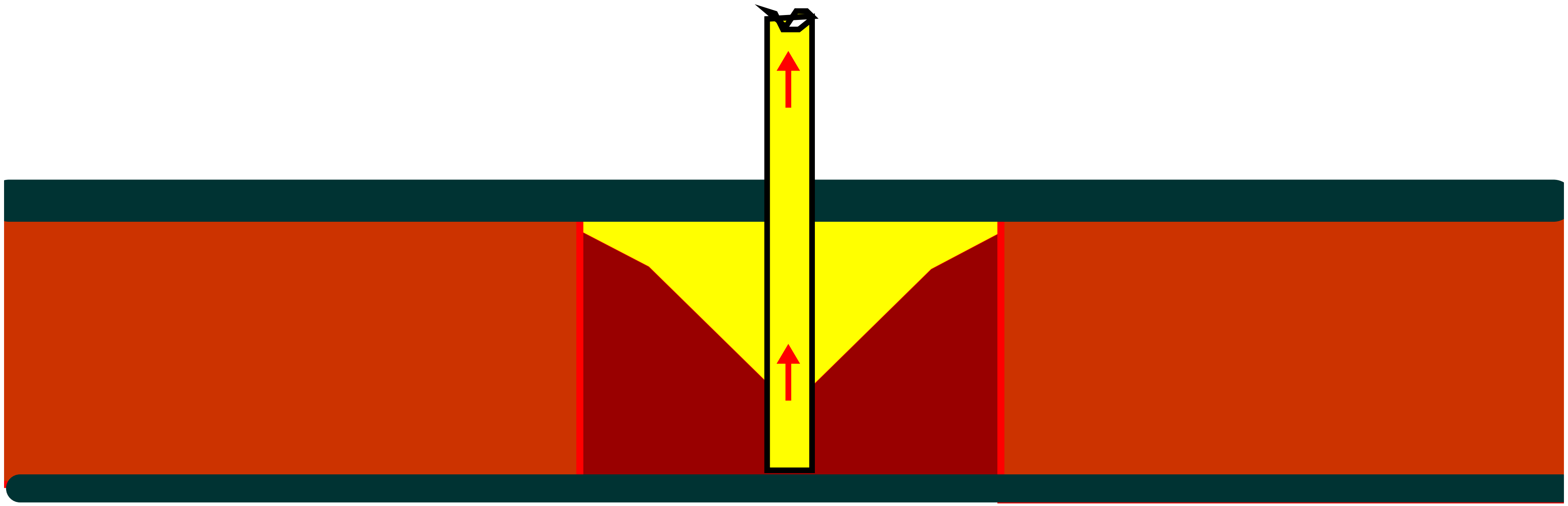

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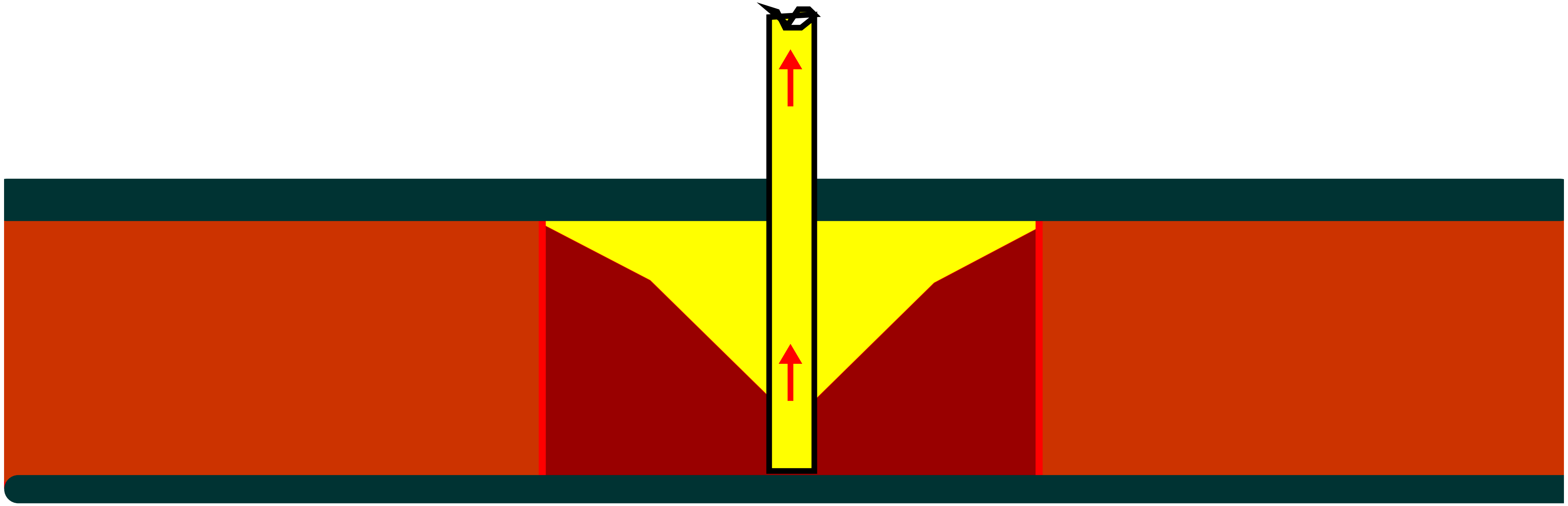

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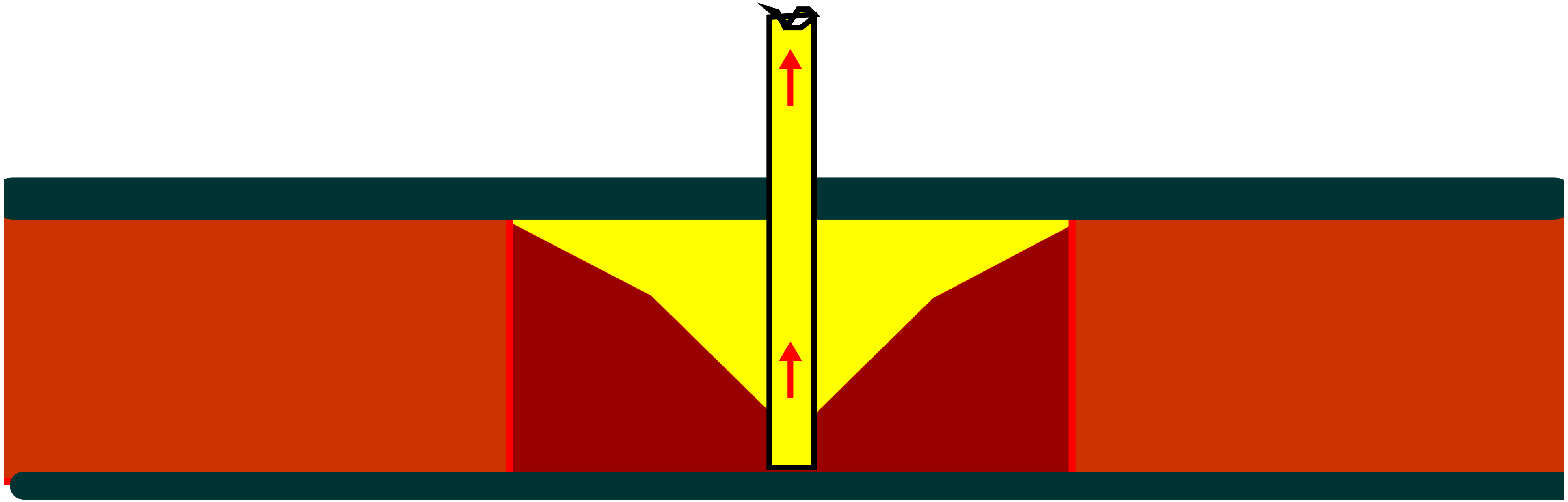

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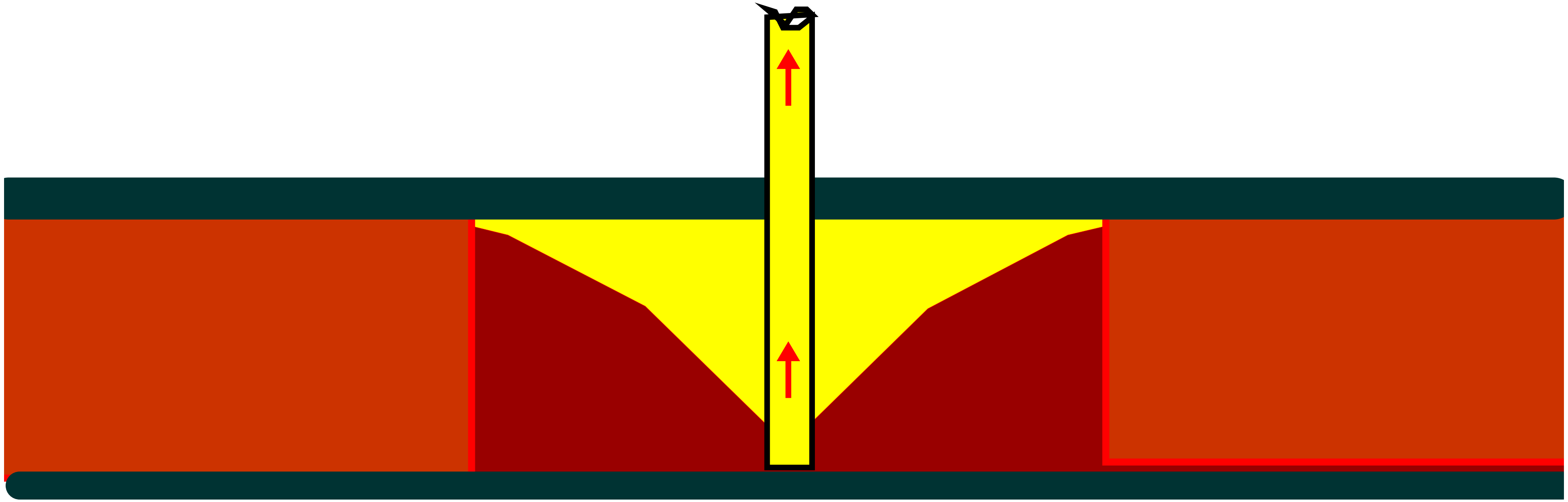

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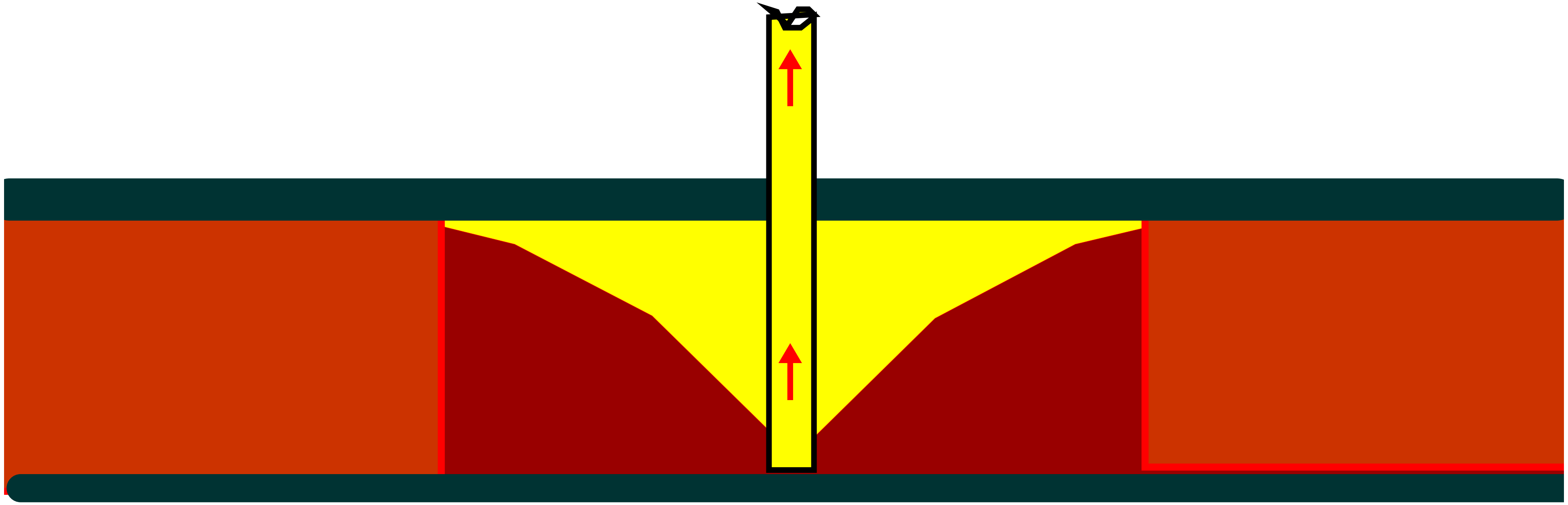

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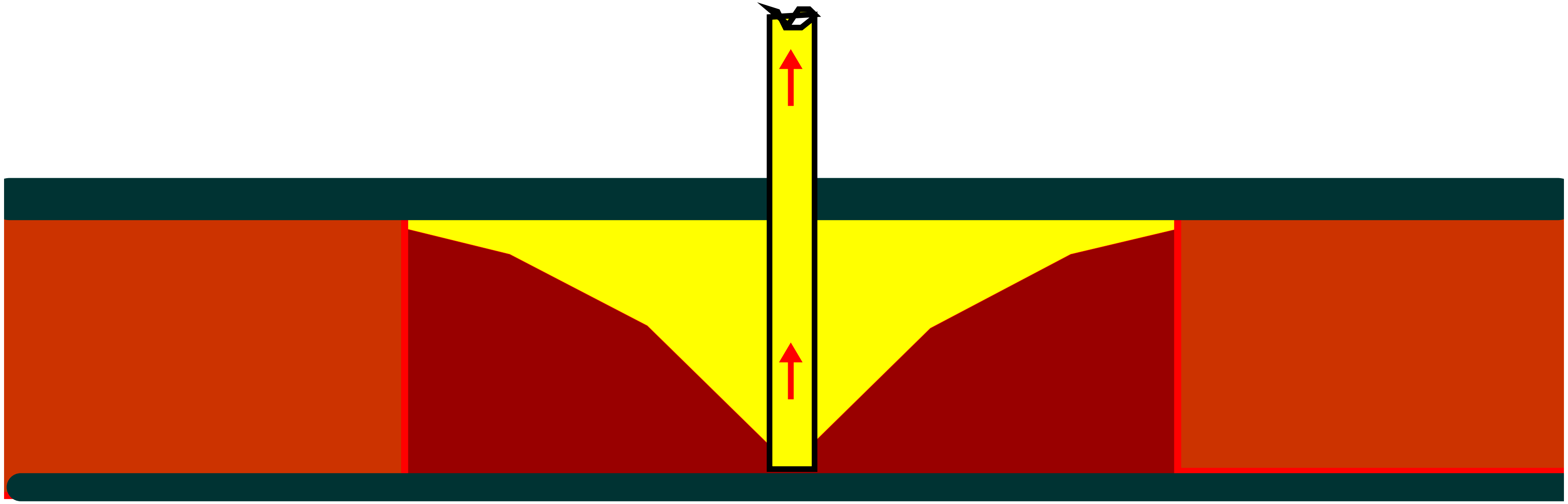

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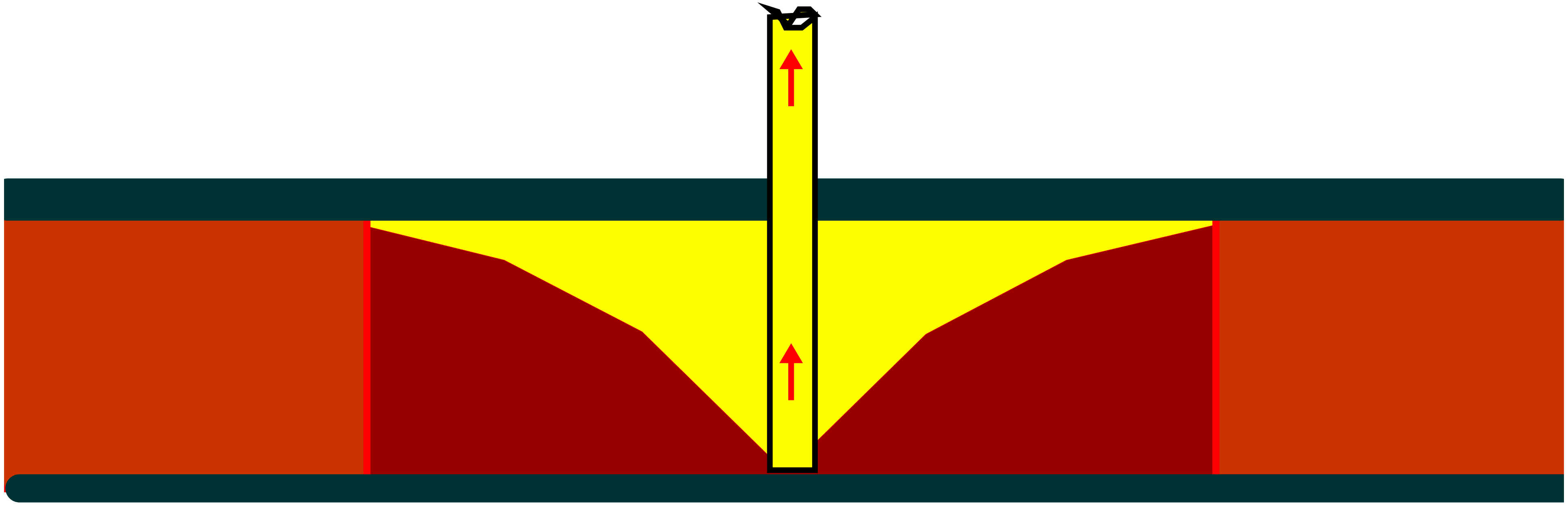
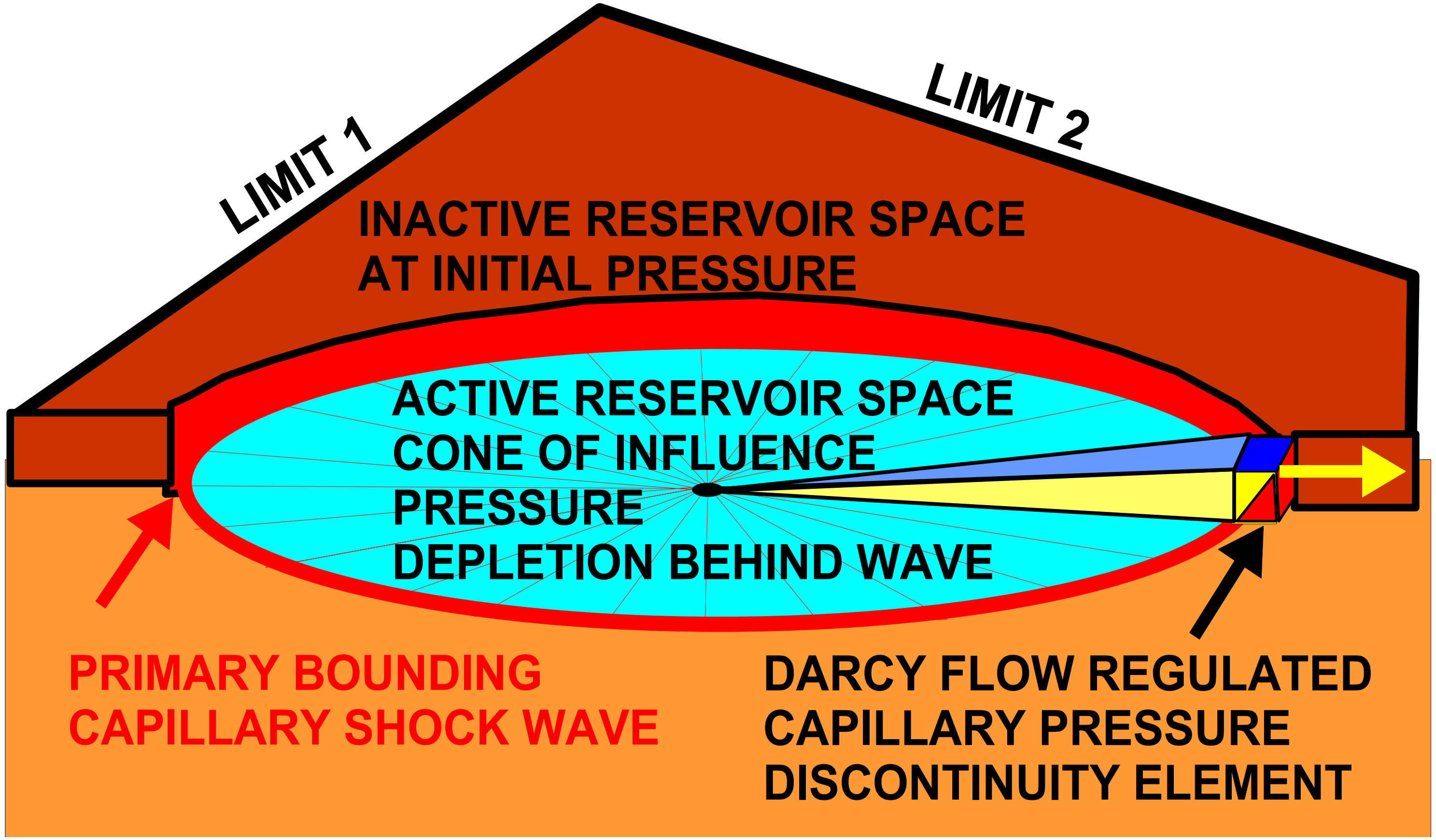
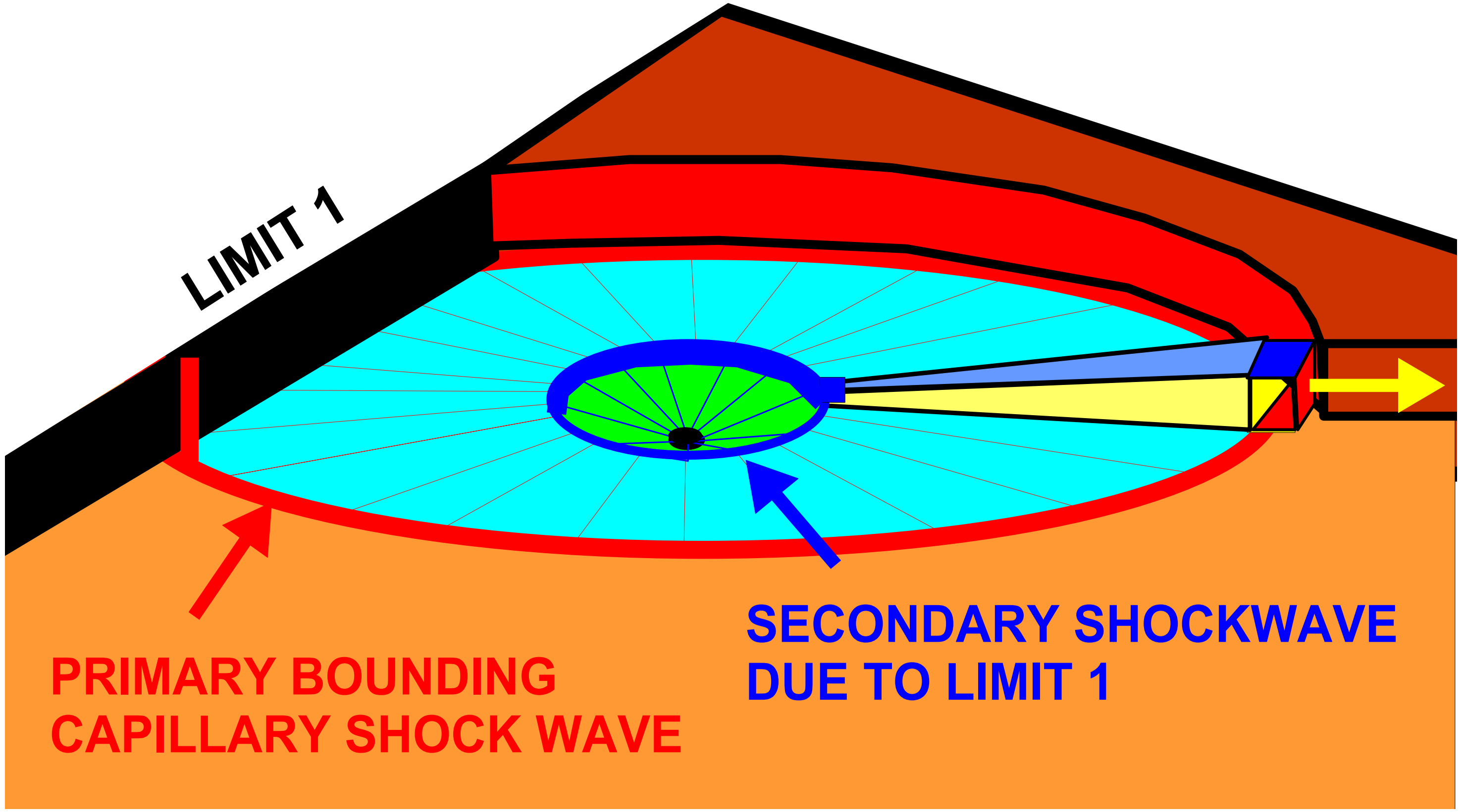


Moving Capillary Shockwave Boundary



Secondary Cone Inside Primary Cone of Influence



The Differences Between the Equations

Classic Diffusivity Equation: $\left(\frac{\delta^2 P}{\delta x^2}\right) = \left(\frac{1}{\eta}\right) * \left(\frac{\delta P}{\delta t}\right) + 0$

Proposed Diffusivity Equation: $\left(\frac{\delta^2 P}{\delta x^2}\right) = \left(\frac{1}{\eta}\right) * \left(\frac{\delta P}{\delta t}\right) + \rho * \phi^2 * A * C_t * \left(\frac{\delta^2 P}{\delta t^2}\right)$

Total System Compressibility

Acceleration Constant

Porosity

Density

Inertial Term – 2nd Order Partial with P & t

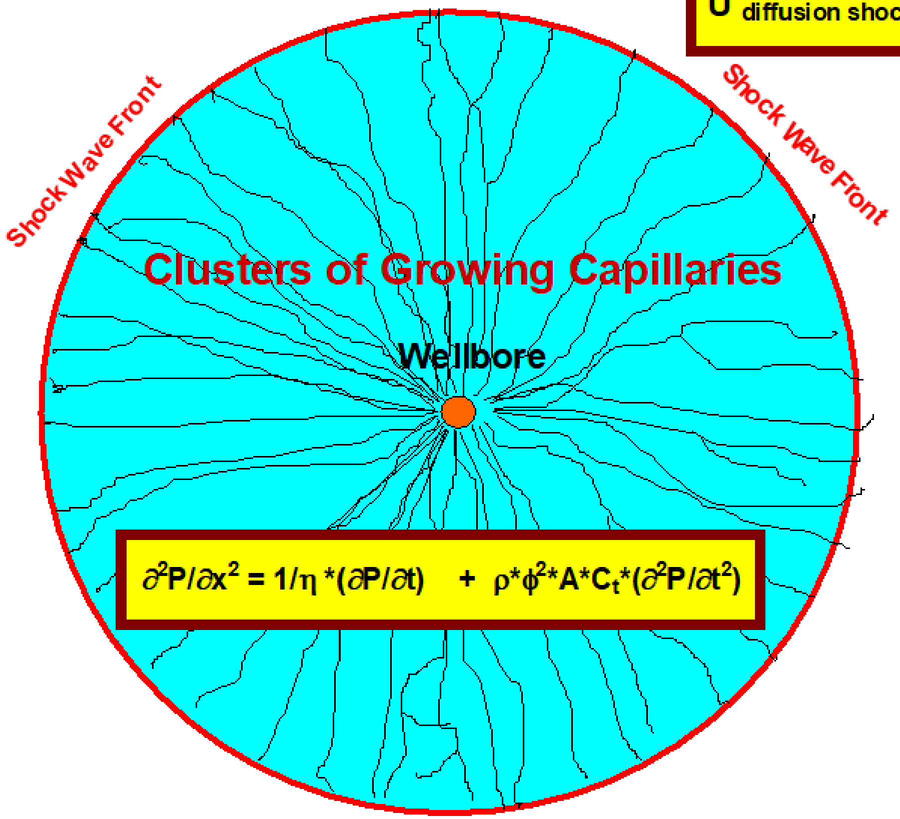
Consequences of the Inclusion of Inertia, Capillary Threshold Pressure and 2nd Law of Thermo Constraints

- Real, Physical Capillaries
- Physical Shock Front as Primary Boundary Condition
- Shock Fronts Cannot Bounce/Reflect; Must Regenerate
- Pressure Responses During Boundary Dominated Radial Flow Regimes are Log-Linear

- Cannot flow fluid out of a pore until the pore throat has ruptured
- A pressure transient is not faster than the speed of light

Solution to the Diffusivity Equation, Including Inertia

$U_{\text{diffusion shockwave}} = (\eta/t)^{1/2}$



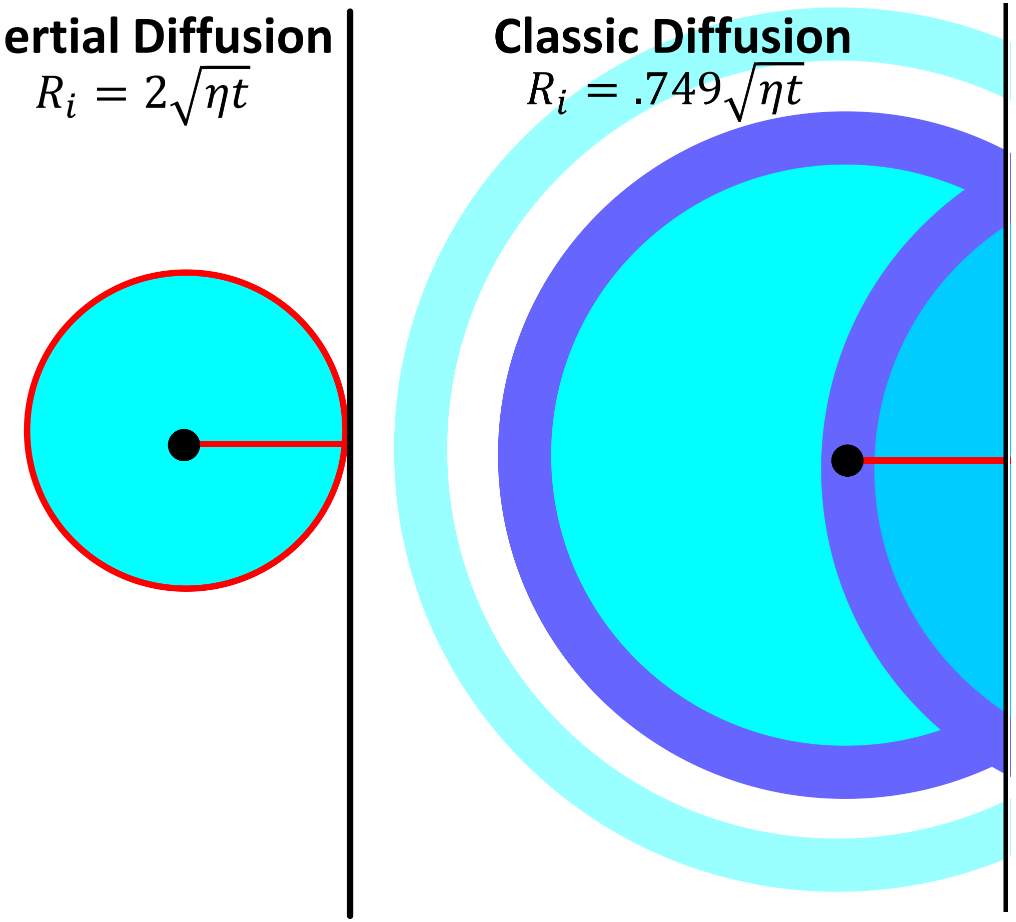
Comparison of Boundary Responses

Inertial Diffusion

$R_i = 2\sqrt{\eta t}$

Classic Diffusion

$R_i = .749\sqrt{\eta t}$



Inertia-Included Diffusivity Solution In Oilfield Units

$$P(r,t) = P_i + 70.6\frac{q * \mu * B}{kh} * \frac{M_n}{M_0} * \frac{t_1}{2.25} * ln\left(-\frac{1.781 * r^2}{4 * \eta * t}\right) + \Delta P(r,t)|_{n-1}$$

$$R_n = 2\sqrt{(\eta * t_{1hr} * \frac{t}{t_n})}$$

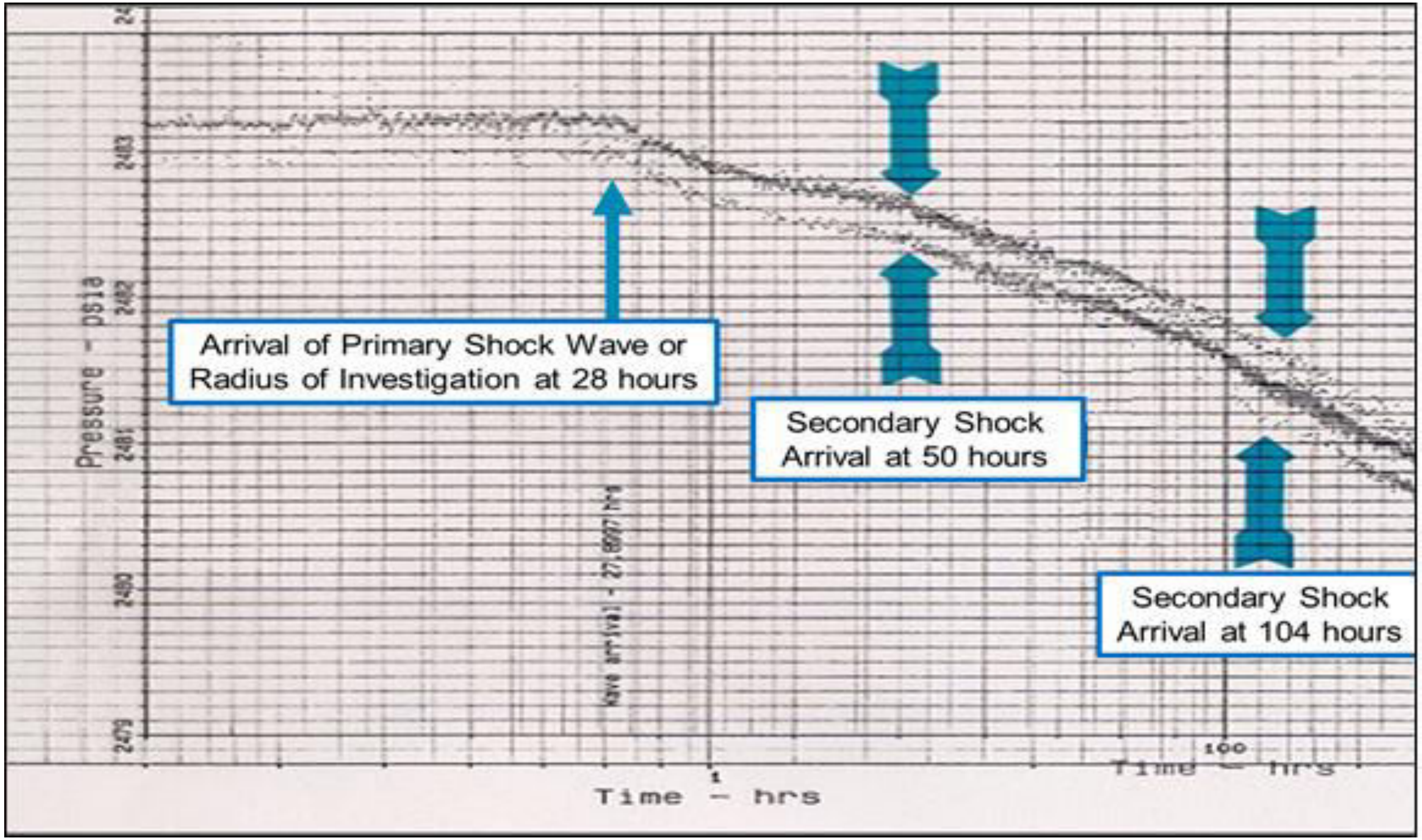
$$\Delta P(r,t)|_{n-1} = P(r,t)|_{n-1} - P(r,t)|_n - \Delta Pcap$$

$$P(r,t) = P(r,t)\ evaluated\ at\ the\ greatest\ r_n\ that\ has\ passed\ that\ point$$

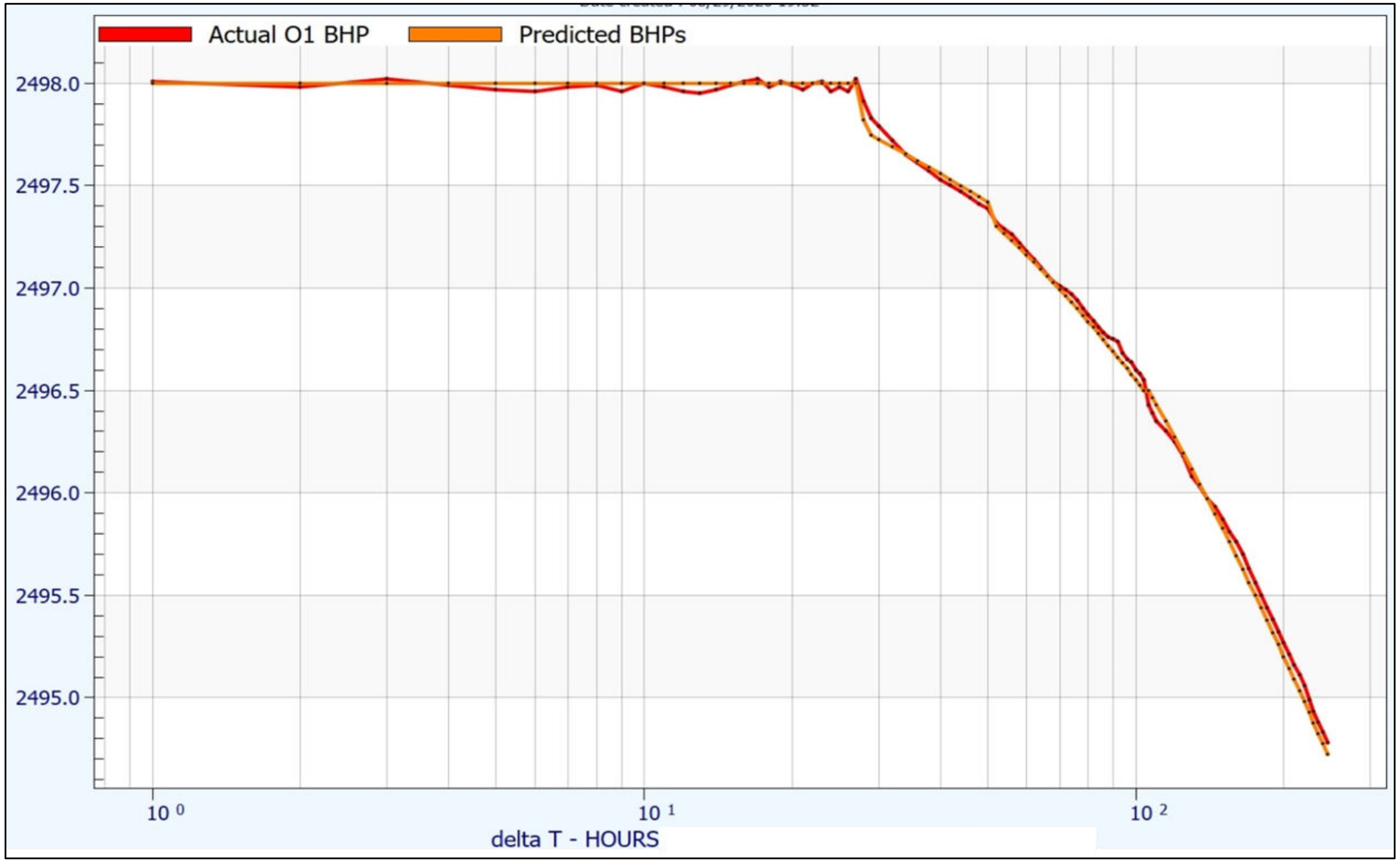
Case Study – Interference in Observation Well 2000 feet away

Parameter	Value	Units
Pi (Initial Reservoir Pressure)	2500	psia
Reservoir Temperature	170.0	Degrees F
R_w (wellbore radius)	0.35	feet
ϕ	0.28	fraction
Gas Gravity	0.600	dimensionless
Z-factor	0.877	dimensionless
Gas Density at Reservoir T, P	7.3	lbm/ft^3
μ	0.018	cp
Sg	0.8	fraction
Sw	0.2	fraction
Bg	1.118	Res bbl/MCF
Ct	3.29E-04	1/psi
Cg	3.92E-04	1/psi
Cw	3.00E-06	1/psi
Cf	1.50E-05	1/psi
MTS (mid-time slope)	1.10	psi/cycle
O1 Flow Rate	0	Mscf/D
P1 Flow Rate	17000	Mscf/D
net pay	130	feet
Distance between wells	2000	feet

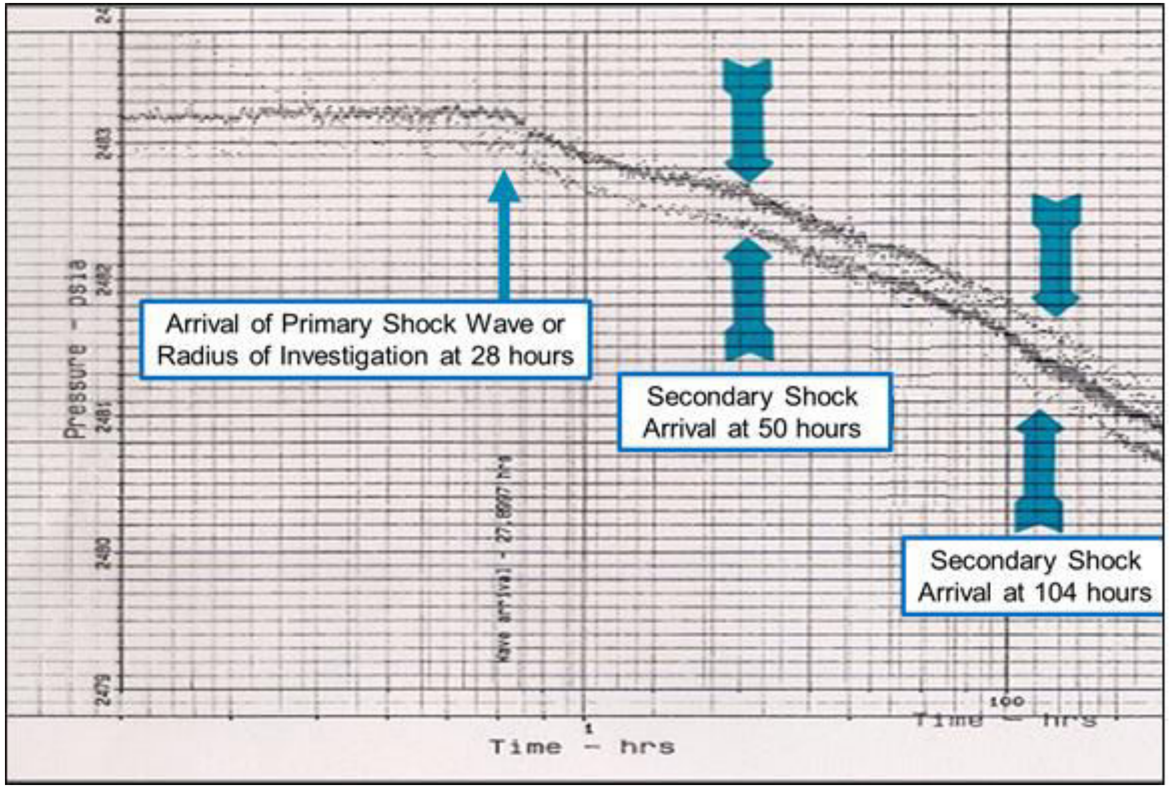
Observation Well Data



Actual vs. Calculated BHPs Using Inertia-Included Diffusivity Eqn.

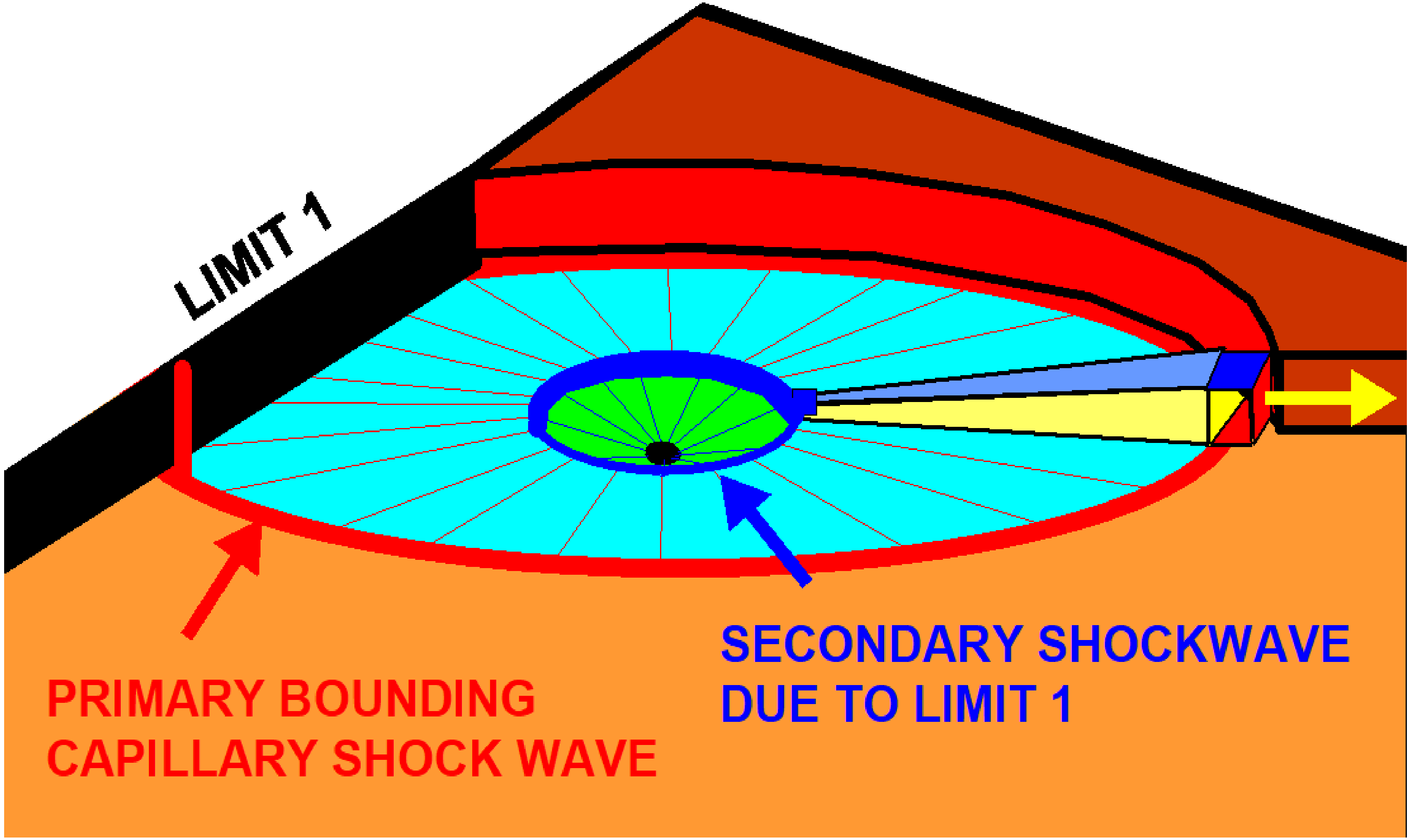


Predicted and Actual Arrival Times



Parameter	Distance from Prod (feet)	Time (hour)
Ri arrival	2000	28.0
R1 arrival	2000	50.5
R2 arrival	2000	103.8

Better Physics = Better Results



Conclusions

- The inclusion of inertia in the solution to the diffusivity equation is required to impose a second law thermodynamic constraint on the active reservoir
- The inclusion of capillary threshold pressure at each rupturing pore throat provides a moving boundary condition
    - It also allows for a capillary structure with finite wall strengths

- Through continuity, the energy equation (first law of thermodynamics) and the inertial wave equations derived in the paper (second law of thermodynamics' constraint), a more rigorous physical and mathematical model has been developed to explain and predict the behavior of transient flow at any point in the reservoir

Future Work by the Authors

1. The end of the effective drainage area in tight/unconventional reservoirs
2. Well test planning with shock fronts
3. The transition period between the end of transient flow and the beginning of some form of steady state flow
4. The recognition of boundary contact types (fault, strat, water contact, etc.)